Optimización Acelerada del Flujo de Potencia Mediante Técnicas de Aproximación Lineal

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Resumen: La resolución de problemas de flujo de potencia óptimo (OPF) se hace cada vez más compleja a medida que aumenta el tamaño del sistema eléctrico. Para hacer frente a esta situación, se emplean diversas técnicas para acelerar el proceso de solución. Un método comúnmente utilizado es la aproximación DC, que simplifica el problema, pero conlleva una cierta pérdida de información. Este trabajo explora la implementación de un algoritmo diseñado para optimizar los cálculos OPF mediante el uso de Aproximaciones Lineales Sucesivas (LSA), basado en el Modelo de Inyección en Bus (BIM) para el análisis del flujo de potencia. Este enfoque comienza con un modelo de relajación convexo, que luego se transforma en una serie de LSA, lo que permite un cálculo más rápido al tiempo que produce soluciones aceptables. Durante la ejecución del algoritmo, se seleccionan datos esenciales del sistema eléctrico estudiado, se evalúan y se comparan con los resultados obtenidos al aplicar el OPF clásico. Las soluciones producidas por el enfoque LSA satisfacen las restricciones operativas del sistema a la vez que reducen significativamente el tiempo de cálculo. Se puede concluir que, a medida que aumenta el número de buses, también crece la cantidad de variables y el tiempo de resolución. En este sentido, la aplicación del método LSA permite reducir los tiempos en más de un 10.00 %, manteniendo un error en el cálculo de la potencia por debajo del 7,00 %.

Palabras clave: Sistema de Eléctricos de Potencia, Aproximaciones lineales Sucesivas, Flujo Óptimo de Potencia, Optimización

Accelerated Power Flow Optimization Through Linear Approximation Techniques

Abstract: Solving Optimal Power Flow (OPF) problems, becomes increasingly complex as the size of the electrical system grows. To address this, various techniques are employed to accelerate the solution process. One commonly used method is DC approximation, which simplifies the problem but leads to a certain loss of information. This paper explores the implementation of an algorithm designed to optimize OPF calculations using Successive Linear Approximations (LSA), based on the Bus Injection Model (BIM) for power flow analysis. This approach starts with a convex relaxation model, which is then transformed into a series of LSA, enabling faster computation while yielding acceptable solutions. During the execution of the algorithm, essential data from the studied electrical system are selected, evaluated, and compared with the results obtained from applying the classic OPF. The solutions produced by the LSA approach meet the operational constraints of the system while significantly reducing computation time. It can be concluded that as the number of buses increases, the number of variables and resolution time also increase. In this regard, applying the LSA method allows for a reduction in resolution time by more than 10.00 %, while maintaining a power calculation error below 7.00 %.

Keywords: Electric Power System, Linear Successive Approximations, Optimal Power Flow, Optimization

1. INTRODUCTION

The continuous growth in demand requires the expansion of electrical power systems and their interconnections to warrant a consistent and high-quality supply. However, this expansion introduces various challenges due to load variability, voltage levels, and the changeability of components, among other factors. Efficient system management is a constant concern, making the analysis of Optimal Power Flow (OPF) critically important (Buason et al., 2022).

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The ongoing expansion of Electrical Power Systems, coupled with their interconnection to guarantee a continuous and reliable supply, presents several challenges. Efficient management of these systems is crucial and analyzing Optimal Power Flow (OPF) is a task of vital importance (Frank & Rebennack, 2016).

On the other hand, Buason et al. (2022) present a Conservative Linear Approximation (CLA) that adapts to the system and operational range relevant to achieving optimal performance with respect to an error metric. Additionally, they apply the CLA to a variety of optimal power flow test cases (Chowdhury et al., 2023).

OPF is a fundamental technique in the real-time operation of Electrical Power Systems (EPS), particularly in the smart grid context, where both power consumption and generation fluctuate significantly over time (Montoya-Giraldo et al., 2017). Zhao et al. (2022) propose a linear model for determining losses, referred to as the Line Loss Outer Approximation (LLOA), which implements an approximation for calculating losses.

Meanwhile, Pesántez et al. (2024) suggest that electrical system planning involves evaluating short-, medium-, and long-term perspectives, each encompassing distinct characteristics and facing specific challenges.
OPF involves the strategic planning of generator dispatch to optimize a specific objective function while adhering to various constraints. The objective function may focus on minimizing system losses, reducing generation costs, or achieving other operational goals. By considering factors such as power demand, system capacity, and network stability, OPF ensures efficient and cost-effective operation of electrical power systems.

Through OPF, operators can maintain reliability and efficiency in real-time power system management (Pareja, 2008). The OPF problem was originally addressed in the early 1960s as an extension of the economic dispatch problem (Muñoz & Quezada, 2015). In this context, OPF is expected to achieve, among other things, the following objectives:

- Ensure operational safety.
- Identify an economically profitable point of operation.

This is accomplished by minimizing an objective function and adjusting various control parameters while accounting for equality and inequality constraints. These constraints are utilized to represent active power balance and various operational limits (Frank & Rebennack, 2016).

Various methodologies exist for solving the OPF problem, depending on the simplifications considered. In this context, Abdi et al. (2017) classify the OPF based on:

- Objective function: Determined by what you aim to optimize, either the cost or performance of the system.
- Constraints: These could be either equality or inequality constraints.
- Network Model: The most common types are AC, DC, and Hybrid models.
- Linearization.
- Temporal analysis: It varies depending on whether the system is stationary or dynamic.
- Solution Techniques: These can either be exact or approximate.
- Security Constraints.
- Deterministic or Stochastic methodologies.
- Probabilistic.

The contribution generated by this paper is summarized as follows:

- Validation of the LSA algorithm's accuracy.
- Implementation of LSA to reduce computational times in Ecuador's SNI.

However, the primary limitation identified in this work relates to the accuracy of the data used to develop the SIN, as some of it was sourced from literature reviews. This may lead to potential discrepancies when compared to other established models.

1.1. Bus Injection Model (BIM)

The BIM model is employed, among other issues, for OPF models with a semi-defined scheduling approach for electrical networks. Figure 1 illustrates a group of busbars directly linked to the busbar i. To specify the principle of using BIM, it must be considered that:

S_i^{gen} = Power flowing toward the bus i.
S_i^{load} = Power demand flowing the bus i.
S_i = Power Injected by the bus i.
S_i^{trans} = Power flowing between two adjacent buses.

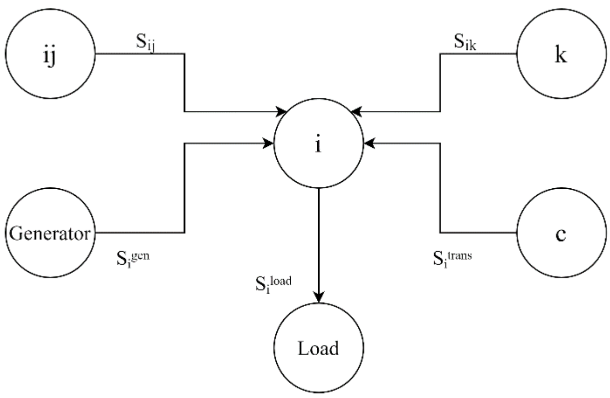

Figure 1. Power balance and magnitudes in a bus associated with contiguous busbars (Aigner et al., 2023)

BIM is widely utilized to establish the active and reactive power injection at every bus. This approach is commonly used for the OPF (Usman et al., 2020).

1.2. Alternating Current Power Flow Optimization (AC-OPF)

AC-OPF serves as a crucial tool enabling power system operators to make precise decisions in managing the electric power systems, while also considering economic factors affecting the market. Its primary role is to maximize the efficient use of resources while safeguarding the interests of all stakeholders involved in the energy market (Aigner et al., 2023).

Typically, the AC-OPF algorithm is formulated based on power balance, incorporating voltage and power equations. According to Rahman et al. (2021) the formulation of AC-OPF.

$$minimize\ losses = \sum_{\forall ij \in \Omega L} P_{pa_{ij}} + \sum_{\forall ij \in \Omega L} P_{de_{ij}} \quad (1)$$

Subject to

$$P_i^g + gsh_i * V_i^2 - P_i^d - \sum_{\forall ij \in OL} P_{de_{ij}} - \sum_{\forall ji \in OL} P_{pa_{ij}} = 0 \quad ; \forall i \in OB \quad (2)$$

$$Q_i^g + bsh_i * V_i^2 - Q_i^d - \sum_{\forall ij \in OL} Q_{de_{ij}} - \sum_{\forall ji \in OL} Q_{pa_{ij}} = 0 \quad ; \forall i \in OB \quad (3)$$

$$-th_{max} \leq th_i \leq th_{max} \quad ; \forall i \in OB \quad (4)$$

$$V_{min\ i} \leq V_i \leq V_{max\ i} \quad ; \forall i \in OB \quad (5)$$

$$P_{g\ min\ i} \leq P_{g\ i} \leq P_{g\ max\ i} \quad ; \forall i \in OB \quad (6)$$

$$Q_{g\ min\ i} \leq Q_{g\ i} \leq Q_{g\ max\ i} \quad ; \forall i \in OB \quad (7)$$

$$P_{ij}^2 + Q_{ij}^2 \leq Smax_{ij}^2 \quad ; \forall ij \in OL \quad (8)$$

Where the active and reactive power flows connecting busbars "i", and "j" can be obtained according to the equations (9) and (10) respectably

$$P_{ij} = g_{ij} * a_{ij}^2 * V_i^2 - a_{ij} * V_i * V_j * g_{ij} * \cos\left(\left(th_i - th_j\right) + fi_{ji}\right) - a_{ij} * V_i * V_j * b_{ij} * \text{sen}\left(\left(th_i - th_j\right) + fi_{ji}\right) \quad ; \forall ij \in OL \quad (9)$$

$$Q_{ij} = -\left(b_{ji} + bshl_{ji}\right) * a_{ij}^2 * V_i^2 - a_{ij} * V_i * V_j * g_{ij} * \text{sen}\left(\left(th_i - th_j\right) + fi_{ji}\right) + a_{ij} * V_i * V_j * b_{ij} * \cos\left(\left(th_i - th_j\right) + fi_{ji}\right) \quad ; \forall ij \in OL \quad (10)$$

The primary objective of OPF is to reduce losses in the power system, as demonstrated in equation (1). It aims to identify the optimal combination of control variables, such as power generation and transformer configuration, to minimize energy losses and optimize resource utilization.

Equations (2) and (3) establish two equality constraints related to the balance of active and reactive powers at each node, ensuring that the sum of generated and consumed powers is equal, thus maintaining the system power balance. Equation (4) pertains to the restriction of the phase angle at the busbars. Additionally, equations (5), (6), and (7) set the limits for output voltage magnitude, active power, and reactive power of the generator, correspondingly. Finally, equation (8) defines the maximum allowable power flows, specifying the safe and efficient operating ranges for generators. Equations (9), and (10) provide a general representation of the BIM.

The power flow in the lines, and consequently the system losses, will be determined by trigonometric equations. In this context, the Linear Successive Approximations (LSA) methodology is applied. LSA is a mathematical and computational technique used to obtain a numerical approximation of a complex function by constructing simpler linear functions. This technique is based on the premise that a nonlinear equation can be transformed into a linear equation by introducing an auxiliary function. By using successive approximations, it starts with an initial guesstimate of the solution and, through the auxiliary function, generates a new estimate. This approach allows for an approximate solution to the original problem while significantly reducing the associated computational costs (Javier et al., 2019).
This approach acts as a foundation for implementing other techniques that address a wide range of problems related to optimization and solution finding. According to Javier et al. (2019), a general classification is shown in

Table 1., which provides an overview of the alternatives to consider when making an approximation.

Table 1. Techniques for Optimization and Solution Finding

Approach	Idea
Approximation	Replace a nonlinear function with a linear function
Linear	Find an approximation to the nonlinear function
Successive	Use an iterative approach with sequential steps to find the best solution

In summary, this process involves transforming the nonlinear equations of the OPF problem to discover the optimal operating points for electrical network elements, such as bus voltages and generated powers. The purpose is to minimize power losses in the system (Escudero & Carrión, 2018).

In this context, LSA is expected to converge more quickly, making its computational cost and processing times suitable for application to larger and more complex systems. However, it is important to consider that as the electrical system expands and approximation methods are applied, it may become susceptible to rounding errors, necessitating additional effort in selecting the initial starting point for the solution.

Table 2 provides an overview of four approaches for performing linear approximations. These approaches encompass various strategies to expedite calculations, ranging from the simplest DC approximation to convex relaxations (Fortenbacher & Demiray, 2019).

Table 2. Approximate Linear Approaches Addressing OPF (Yang et al., 2017b)

Approach	Relationship	Approach
DC Power Flows	A linear relationship among active power (P) and phase angle (θ).	DC Power Flows
Taylor Series	Uses Taylor polynomials	Taylor Series
Linear Approximation	Linearizes the AC-OPF equations around a feasible operating point.	Linear Approximation
Convex Relaxation	Converts non-convex relaxations to convex ones.	Convex Relaxation

2. METHODOLOGY

To analyze the incorporation of LSA, the methodology proposes three stages: starting with a general description of the process, followed by determining the solvers used for the solution of the algorithm, and concluding with a case study of the national interconnected system.
Figure 2 provides a simplified overview of the procedure used for this process. It begins by applying mathematical approximations to the model, then considers the independent variables, and finally approximates values that are close to unity. Each step is detailed lower.

2.1. Linear Successive Approximations

To develop a system that uses successive linear approximations in the OPF problem, we start with a nonlinear AC power flow model. This nonlinear model serves as the foundation for the OPF LSA method, which involves gradually approximating the nonlinear model through a series of linear equations.

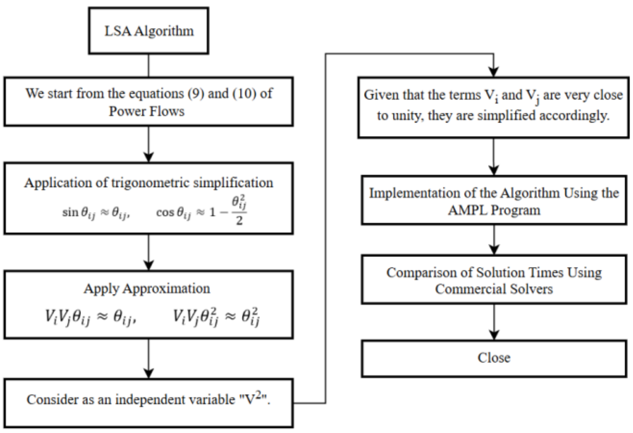

Figure 2. Basic methodology for implementing the algorithm

Based on equations (9) and (10) for power flows in branches, which are primary sources of non-convexity, and considering the works by Yang et al. (2017b, 2018), the simplification of sinusoidal functions is applied, as shown in equation (11).

$$\sin\theta_{ij} \approx \theta_{ij}, \qquad \cos\theta_{ij} \approx 1 - \frac{\theta_{ij}^2}{2} \tag{11}$$

By replacing equations (11) into equations (9) and (10), the resulting expressions are found:

$$P_{ij} = g_{ij} * a_{ij}^2 * V_i^2 - a_{ij} * V_i * V_j * g_{ij} * 1 - \frac{\theta_{ij}^2}{2} - a_{ij} * V_i * V_j * b_{ij} * \theta_{ij} \tag{12}$$

$$Q_{ij} = -\left(b_{ji} + bshl_{ji}\right) * a_{ij}^2 * V_i^2 - a_{ij} * V_i * V_j * g_{ij} * \text{sen}\,\theta_{ij} + a_{ij} * V_i * V_j * b_{ij} * 1 - \frac{\theta_{ij}^2}{2} \tag{13}$$

We can observe a direct relationship between V and θ, thus the following considerations are made:

$$V_iV_j\theta_{ij} \approx \theta_{ij}, \qquad V_iV_j\theta_{ij}^2 \approx \theta_{ij}^2 \tag{14}$$

The approximations provided in equation (14) are applied in the DC network model. This approach is taken because the voltage magnitudes V_i or V_j are generally close to $1.0\,p.u.$, which makes the θ_{ij} (Yang et al., 2018).

During the derivation phase, both linear and quadratic approximations are preserved. The linear approximations enhance the representation of the DC network representation, while the quadratic approximations account for network losses. To accomplish this, the nonlinear term V_iV_j is transformed into a linear term through a mathematical process, as demonstrated in the equation (15) (Yang et al., 2017a).

$$V_iV_j = \frac{V_i^2 + V_j^2}{2} - \frac{\left(V_i - V_j\right)^2}{2} \tag{15}$$

In equation (15), the first term becomes linear by considering V^2 as the independent variable, whereas the second term illustrates the impact of voltage on losses. This approximation assumes that the values of V_i and, V_j are closed to $1.0\,p.u.$ (Yang et al., 2017b).

$$\frac{\left(V_i - V_j\right)^2}{2} \approx \frac{1}{2}\left[\left(V_i - V_j\right) * \frac{V_i + V_j}{2}\right]^2 = \frac{\left(V_i^2 - V_j^2\right)^2}{8} \tag{16}$$

By substituting equation (16) into equation (15), the expression for the term V_iV_j is adjusted as follow:

$$V_iV_j \approx \frac{V_i^2 + V_j^2}{2} - \frac{\left(V_i^2 - V_j^2\right)^2}{8} \tag{17}$$

By replacing equations (14) and (17) interested in equations (12) and (13) respectively, we obtain the linear approximation shown in the following equations for active and reactive power flow, considering losses, dispatched from bus i. to bus j.

$$P_{de_{ij}} = g_{ij} * a_{ij}^2 * \frac{V_i^2 - V_j^2}{2} - b_{ij} * a_{ij} * \theta_{ij} + a_{ij} * g_{ij} * \tag{18}$$

$$\left[\frac{\theta_{ij}^2}{2}+\frac{\left(V_i^2-V_j^2\right)^2}{8}\right] \quad ;\forall ij \in OL$$

$$Q_{de_{ij}} = -\left(b_{ji}+bshl_{ji}\right)*a_{ij}^2*\frac{V_i^2-V_j^2}{2}-g_{ij}*a_{ij}*\theta_{ij}-b_{ij}*a_{ij}*\left[\frac{\theta_{ij}^2}{2}+\frac{\left(V_i^2-V_j^2\right)^2}{8}\right] \quad ;\forall ij \in OL \tag{19}$$

Furthermore, equations (20) and (21) describe the active and reactive power flows interacting between buses i and j. This information enables the analysis of power transfer between buses, accelerating a more detailed examination of flows within the electrical system (Jha & Dubey, 2021).

$$P_{pa_{ij}} = g_{ij}*a_{ij}^2*\frac{V_j^2-V_i^2}{2}+b_{ij}*a_{ij}*\theta_{ij}+a_{ij}*g_{ij}*\left[\frac{\theta_{ij}^2}{2}+\frac{\left(V_j^2-V_i^2\right)^2}{8}\right] \quad ;\forall ij \in OL \tag{20}$$

$$Q_{pa_{ij}} = -\left(b_{ji}+bshl_{ji}\right)*a_{ij}^2*\frac{V_j^2-V_i^2}{2}+g_{ij}*a_{ij}*\theta_{ij}-b_{ij}*a_{ij}*\left[\frac{\theta_{ij}^2}{2}+\frac{\left(V_j^2-V_i^2\right)^2}{8}\right] \quad ;\forall ij \in OL \tag{21}$$

2.2. Software

The implementation of the algorithm is carried out using the AMPL program, which employs three commercial solvers: KNITRO, IPOPT, and CONOPT. These solvers are specifically chosen for their capability to address nonlinear optimization problems. For all cases, the entire computation time necessary to solve the problem is recorded.

2.3. Ecuadorian Model System

The development of the Ecuadorian system model was based on data provided by the current Master Electricity Plan (PME) (Plan Maestro de Electricidad – Ministerio de Energía y Minas, 2018). In this context, the Ecuadorian SNI outlines the transmission system, though it is represented in a simplified form with a network equivalent for certain regions. This study scenario draws on the work of Robinson et al. (2022), which includes the topological configuration of the 2020 single-line diagram. Figure 3 illustrates the electrical power system.

3. RESULTS

In this section, we present the calculations obtained by applying the proposed LSA algorithm. First, the IEEE 14 Busbars system is used to validate the performance of the algorithm, and subsequently, it is applied to the IEEE PES 793-bus system as well as to the Ecuadorian Electrical System. For comparison purposes, two metrics are employed: the difference in system power losses and the solution time of the algorithm. In all cases, three commercial solvers from the AMPL program are used: KNITRO, IPOPT, and CONOPT.

3.1. Algorithm Validation

In the IEEE 14-bus test system, the AC-OPF algorithm is used as a baseline, and an analysis is conducted by comparing it with the LSA algorithm. The analysis considers voltages and angles at each bus as metrics, as shown in Table 3. Additionally, Table 4 uses system power losses as a metric to evaluate the performance of the algorithm.

Table 3. IEEE 14-bus System Using AC-OPF and LSA

	AC-OPF		LSA	
Bus	**Voltage (V)**	**Angle (th)**	**Voltage (V)**	**Angle (th)**
1	1.060	0.000	1.060	0.000
2	1.058	0.000	1.059	0.000
3	1.056	-0.001	1.057	-0.001
4	1.044	-0.002	1.048	-0.006
5	1.043	0.000	1.048	-0.004
6	1.062	0.010	1.031	0.009
7	1.056	0.066	1.041	0.063
8	1.100	0.209	1.100	0.221
9	1.033	0.008	1.011	0.001
10	1.031	0.003	1.007	-0.003
11	1.043	0.005	1.015	0.000
12	1.046	-0.004	1.015	-0.007
13	1.040	-0.005	1.010	-0.008
14	1.018	-0.016	0.992	-0.022

Table 4. Power Loss System IEEE 14 Busbars

	Solver		
Algorithms	**KNITRO**	**IPOPT**	**CONOPT**
AC-OPF (MW)	0.7548	0.7548	0.7548
LSA (MW)	0.6997	0.6997	0.6997
Error (%)	7.3105	7.3106	7.3106

For power losses, it was observed that calculation errors did not exceed 7.31% for the commercial software. Additionally, in Table 3 the voltage magnitudes showed differences of less than 3.00 %, with similar results for the angle values.

Furthermore, Table 5 exhibits the total solution times using three commercial solvers for AC-OPF and LSA. LSA achieves a notable 20.00 % reduction in solution time compared to AC-OPF under the worst-case conditions.

This validation underscores LSA's capability to deliver satisfactory results while significantly improving solution times for the problem at hand.

3.2. IEEE PES 793-bus System

Considering the IEEE PES 793-bus test system presented by Abdi et al. (2017), the case files are available under a Creative

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Table 6 presents the power losses, showing that the difference in response does not exceed 7.14 % compared to the AC-OPF algorithm, irrespective of the solver used.

Table 5. Resolution times of AC-OPF and LSA Algorithms IEEE 14 Busbars System

	Total resolution time		
Algorithms	**KNITRO**	**IPOPT**	**CONOPT**
AC-OPF	0.078 s	0.047 s	0.016 s
LSA	0.063 s	0.016 s	0.001 s
Difference (%)	20.000 %	66.670 %	42.400 %

Table 6. Power Loss System IEEE PES 793-bus system

	Solver		
Algorithms	**KNITRO**	**IPOPT**	**CONOPT**
AC-OPF (MW)	53.623	53.623	53.623
LSA (MW)	57.471	57.471	57.471
Error (%)	7.175	7.174	7.175

Similar to the IEEE 14-bus System case, when evaluating the total system resolution time using the LSA algorithm and comparing it to the AC-OPF algorithm with the three commercial solvers: KNITRO, IPOPT, and CONOPT, the results indicate a reduction in computation times by 20.00 %, 18.18 %, and 24.25 %, respectively.

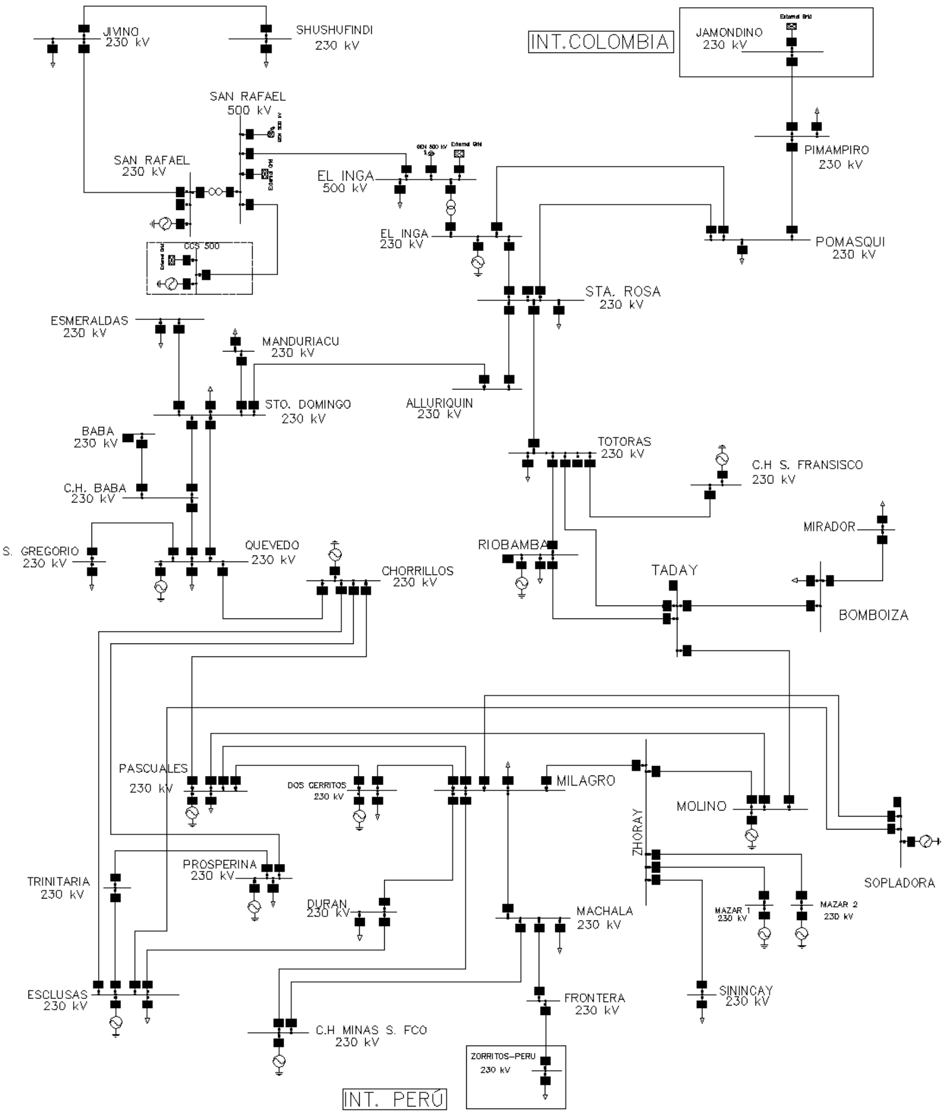
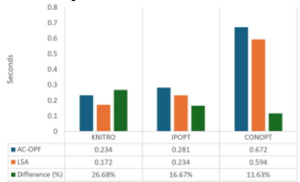


Figure 3. National Interconnection System (SNI) of Ecuador (Robinson & Acosta, 2022)

3.1 Ecuadorian National System

Figure 4 shows the results of applying the algorithm to the SIN.

a) Power Loss System Ecuadorian National System



b) Resolution times of AC-OPF and LSA Algorithms Ecuadorian National System

Figure 4. Ecuadorian National System Results

The metrics presented include system losses and solution time. When applying the algorithm within the SIN framework, a notable variation of 6.13 % is observed in system loss power across the three solvers. The result for power loss error in the Ecuadorian SIN aligns with findings from the previously analyzed IEEE PES 793 bus test system, where the power loss error reached 7.17 %. Finally, significant reductions in calculation times are highlighted, with improvements of up to 11.62 % compared to the AC-OPF algorithm.

5. CONCLUSIONS

During the development of the LSA model, the convex relaxation method served as a baseline for evaluating resolution times. Results indicated that the successive linear approximation (LSA) offers a 23.80 % faster resolution time compared to the conventional method.

As the system size grows, the accuracy of linear approximations becomes increasingly critical, directly influencing the objective of minimizing losses. For the 14-bus IEEE system, an optimization error of 7.31 % was observed between the conventional method and LSA.

However, for the larger 793-bus system, this error increases to 7.71 % compared to the traditional method, as verified by all three solvers in the AMPL software.

When assessing the LSA method within Ecuador's national power system (SNI), its efficiency in execution time stands out compared to the conventional approach. The LSA system demonstrates significant improvements in resolution time, with variations ranging from 18.18 % to 24.25 % across the three solvers employed. However, it is important to note that while the LSA method shows agility, it also results in a 6.13 % increase in active losses compared to the conventional method due to the inherent linear approximations used during optimization.

As indicated in the development of this document, as the size of the power system analyzed increases, the resolution time becomes longer due to the growing number of variables. This underscores the importance of incorporating methodologies that can partially mitigate the convexity associated with OPF. In this case, the approach led to computational time reductions exceeding 20.00 %, while keeping the error in power loss calculation below 8.00 %.

A comparison of the results obtained through LSA with those achieved using techniques such as semidefinite relaxation, clustering-based methods, or convex hulls would be insightful. This would facilitate an evaluation of the effectiveness of various approaches, considering that the algorithms employed in this study were based on interior-point methods.

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